LESSON 5: Decimals

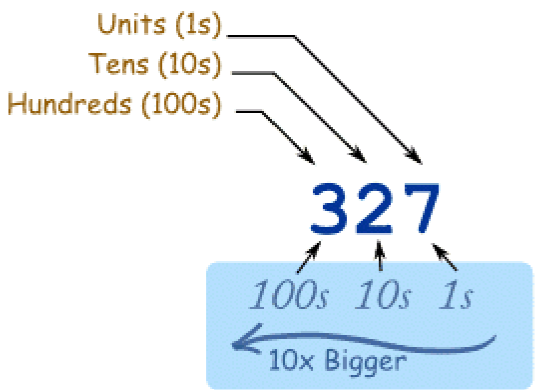
A Decimal Number (*based on the number 10*) contains a **Decimal Point**.

1.- Place Value

To understand decimal numbers you must first know about Place Value.
When we write numbers, the **position** (or "**place**") of each number is important.

In the number 327:

- the "7" is in the **Units** position, meaning just 7 (or 7 "1"s),
- the "2" is in the **Tens** position meaning 2 tens (or twenty),
- and the "3" is in the **Hundreds** position, meaning 3 hundreds.



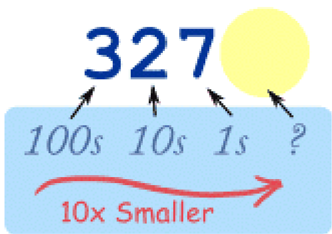
"Three Hundred Twenty Seven"

As we move left, each position is 10 times bigger!

From Units, to Tens, to Hundreds

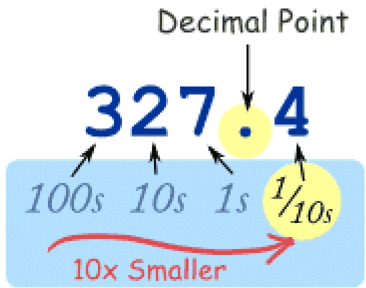
As we move right, each position is 10 times **smaller**.

From Hundreds, to Tens, to Units



But what if we continue past Units?
What is **10 times smaller** than Units?
$^1/_{10}$ ths (Tenths) are!

But we must first write a **decimal point**,so we know exactly where the Units position is:



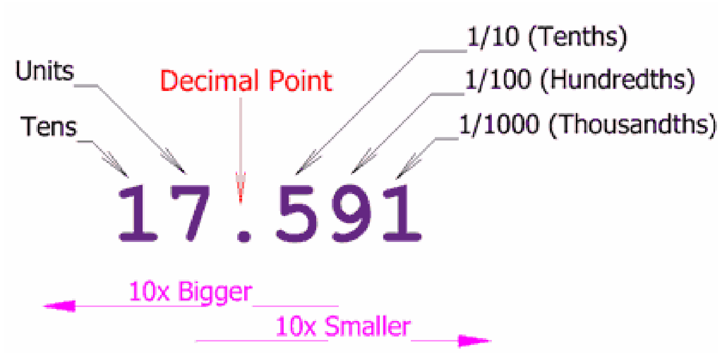
"three hundred twenty seven **and four tenths**"

And that is a Decimal Number!

2.- Decimal Point

The **decimal point** is the most important part of a Decimal Number. It is exactly to the right of the Units position. Without it, we would be lost ... and not know what each position meant.

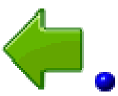
Now we can continue with smaller and smaller values, from **tenths**, to **hundredths**, and so on, like in this example:



3.- Large and Small

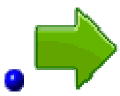
So, our Decimal System lets us write numbers as large or as small as we want, using the decimal point. Numbers can be placed to the left or right of a decimal point, to indicate values greater than one or less than one.

17.591



The number to the left of the decimal point is a whole number (17 for example)

As we move further left, every number place gets **10 times bigger**.



The first digit on the right means **tenths** (1/10).

As we move further right, every number place gets **10 times smaller** (one tenth as big).

4.- Definition of Decimal

The word "Decimal" really means "based on 10" (From Latin *decima*: *a tenth part*). We sometimes say "decimal" when we mean anything to do with our numbering system, but a "Decimal Number" usually means there is a Decimal Point.

You could think of a decimal number as a whole number plus tenths, hundredths, etc:

Example 1: What is 2.3 ?

- On the left side is "2", that is the whole number part.
- The 3 is in the "tenths" position, meaning "3 tenths", or 3/10
- So, 2.3 is "2 and 3 tenths"

Example 2: What is 13.76 ?

- On the left side is "13", that is the whole number part.
- There are two digits on the right side, the 7 is in the "tenths" position, and the 6 is the "hundredths" position
- So, 13.76 is "13 and 7 tenths and 6 hundredths"

Or, you could think of a decimal number as a Decimal Fraction.

A Decimal Fraction is a fraction where the denominator (the bottom number) is a number such as 10, 100, 1000, etc (in other words a power of ten)

So "2.3" would look like this:	23/10
And "13.76" would look like this:	1376/100

Those are all good ways to think of decimal numbers.

5.- Rounding Numbers

What is "Rounding" ?: Rounding means reducing the digits in a number while trying to keep its value similar. *The result is less accurate, but easier to use.*

Example: 73 rounded to the nearest ten is 70, because 73 is closer to 70 than to 80.

Common Method: There are several different methods for rounding, but here we will only look at the common method, the one used by most people.

How to Round Numbers ?:

- Decide which is the last digit to **keep**
- Leave it the same if the **next digit** is less than 5 (this is called *rounding down*)
- But increase it by 1 if the next digit is 5 or more (this is called *rounding up*)

Example: Round 74 to the nearest 10

- We want to keep the "7" as it is in the 10s position
- The next digit is "4" which is less than 5, so no change is needed to "7"

Answer: 70 (74 gets "rounded down")

Example: Round 86 to the nearest 10

- We want to keep the "8"
- The next digit is "6" which is 5 or more, so increase the "8" by 1 to "9"

Answer: 90 (86 gets "rounded up")

So: when the first digit **removed** is 5 or more, increase the last digit **remaining** by 1.

Why does 5 go up ?. 5 is in the middle, so we could go up or down. But we need a method that everyone can agree to always use. *And that is the most important part of the "common" method of rounding.* It is not a perfect method, but it is the one most people use.

Rounding Decimals. First you need to know if you are rounding to tenths, or hundredths, etc. Or maybe to "so many decimal places". That tells you how much of the number will be left when you finish.

Examples	Because ...
3.1416 rounded to hundredths is 3.14	... the next digit (1) is less than 5
1.2635 rounded to tenths is 1.3	... the next digit (6) is 5 or more
1.2635 rounded to 3 decimal places is 1.264	... the next digit (5) is 5 or more

Rounding Whole Numbers: You may want to round to tens, hundreds, etc, In this case you replace the removed digits with zero.

Examples	Because ...
134.9 rounded to tens is 130	... the next digit (4) is less than 5
12,690 rounded to thousands is 13,000	... the next digit (6) is 5 or more
1.239 rounded to units is 1	... the next digit (2) is less than 5

Rounding to Significant Digits: To round "so many" significant digits, just count that many from left to right, and then round off from there. (Note: if there are leading zeros (such as 0.006), don't count them because they are only there to show how small the number is).

Examples	Because ...
1.239 rounded to 3 significant digits is 1.24	... the next digit (9) is 5 or more
134.9 rounded to 1 significant digit is 100	... the next digit (3) is less than 5
0.0165 rounded to 2 significant digits is 0.017	... the next digit (5) is 5 or more

6.- Ordering Decimals

Ordering decimals can be tricky. This is because often we look at 0.42 and 0.402 and say that 0.402 must be bigger because there are more digits.

If you follow the following method you will see which decimals are bigger.

- Set up a table with the decimal place in the same place for each number.
- Put in each number.
- Fill in the empty squares with zeros.
- Compare using the first column, and pick out the highest in order.
- If the digits are equal move to the next column until one number wins.

Example: Order the following decimals: 0.402, 0.42, 0.375, 1.2, 0.85

In a table they will look like this:

Units	Decimal Point	Tenths	Hundredths	Thousandths
0	.	4	0	2
0	.	4	2	0
0	.	3	7	5
1	.	2	0	0
0	.	8	5	0

Compare the Units.	➡	There is a 1, all the rest are 0, so 1.2 must be the highest. (Write it down in your answer and cross it off the table).
Compare the Tenths.	➡	The 8 is highest, so 0.85 is next in value.
There are two numbers with the same "Tenths" value of 4, so move down to the "Hundredths" for the tie breaker	➡	One number has a 2 in the hundredths, and the other has a 0, so the 2 wins. So 0.42 is bigger than 0.402
Go back to comparing the Tenths	➡	0.375 must be next followed by 0.2

The decimals must be in the order, highest to lowest: 1.2, 0.85, 0.42, 0.402, 0.375. Done!

7.- Adding Decimals

To add decimals, follow these steps:

- Write down the numbers, one under the other, with the decimal points lined up.
- Add zeros so the numbers have the same length
- Then add normally, remembering to put the decimal point in the answer

Add 5.3 to 7.04
Line the decimals up, "pad" with zeros and add:

$$\begin{array}{r} 5.3 \\ +7.04 \\ \hline 12.34 \end{array}$$

8.- Subtracting Decimals

To subtract decimals, follow these steps:

- Write down the two numbers, one under the other, with the decimal points lined up.
- Add zeros so the numbers have the same length
- Then subtract normally, remembering to put the decimal point in the answer

Subtract 24.5 from 1.29
Line the decimals up, "pad" with zeros and subtract:

$$\begin{array}{r} 24.50 \\ -\ 1.29 \\ \hline 23.21 \end{array}$$

9.- Multiplying Decimals

How to Multiply Decimals? Just follow these steps:

- Multiply normally, ignoring the decimal points.
- **Then** put the decimal point in the answer - it will have as many decimal places as the two original numbers combined.

In other words, just count up how many numbers are after the decimal point in *both* numbers you are multiplying, then the answer should have that many numbers after *its* decimal point.

5.09 x 7.12 = 36.2408
4.2 x 9,17 = 38.514

10.- Dividing Decimals

To divide a decimal number by a whole number:

- Use Long Division (ignoring the decimal point)
- Then put the decimal point in the same spot as the dividend.

Example: Divide 9.1 by 7
The answer is 1.3

Dividing by a Decimal Number: The trick is to convert the number you are dividing by to a whole number first, by shifting the decimal point of both numbers to the right:

$$6.625 \div 0.53 \rightarrow 662.5 \div 53$$

Now you are **dividing by a whole number**, and can continue as normal.
It is safe to do this if you remember to shift the decimal point of **both numbers** the same number of places.

Example 2: Divide 5.39 by 1.1
You are **not** dividing by a whole number, so you need to move the decimal point so that you **are** dividing by a whole number:

move 1		
5.39	➡	**53.9**
1.1	➡	**11**
move 1		

You are now dividing by a whole number, so you can proceed. Ignore the decimal point and use Long Division:
Put the decimal point in the answer directly above the decimal point in the dividend:
The answer is 4.9

11.- Convert Decimals to Fractions

To convert a Decimal to a Fraction follow these steps:

Step 1: Write down the decimal divided by 1.

Step 2: Multiply both top and bottom by 10 for every number after the decimal point. (For example, if there are two numbers after the decimal, then use 100, if there are three then use 1000, etc.)

Step 3: Simplify (or reduce) the fraction

Example 1: Express 0.75 as a fraction

Step 1: Write down: $\frac{0.75}{1}$

Step 2: Multiply both top and bottom by 100 (becasue there were 2 digits after the decimal place):

× 100

$$\frac{0.75}{1} = \frac{75}{100}$$

× 100

(Do you see how it neatly turns the top number into a whole number?)

-Step 3: Simplify the fraction:

÷ 25

$$\frac{75}{100} = \frac{3}{4}$$

÷ 25

Answer = 3/4

*Note: 75/100 is called a **decimal fraction** and 3/4 is called a **common fraction**)*

Example 2: Express 0.625 as a fraction

Step 1: write down: $\frac{0.625}{1}$

Step 2: multiply both top and bottom by 1000 (there were 3 digits after the decimal place so that is 10×10×10=1000)

$$\frac{625}{1000}$$

Step 3: Simplify the fraction (it took me two steps here):

÷ 25 ÷ 5

$$\frac{625}{1000} = \frac{25}{40} = \frac{5}{8}$$

÷ 25 ÷ 5

Answer = 5/8

Example 3: Express 0.333 as a fraction

Step 1: Write down: $\frac{0.333}{1}$

Step 2: Multiply both top and bottom by 1000 (there were 3 digits after the decimal place so that is 10×10×10=1000) $\frac{333}{1000}$

Step 3: Simplify Fraction:

Can't get any simpler!
Answer = 333/1000

12.- Convert Fractions to Decimals

The simplest method is just divide the top of the fraction by the bottom.

Example 1: Express 3/4 as a Decimal

Step 1: We can mulitply 4 by 25 to become 100

Step 2: Multiply top and bottom by 25:

$$\frac{3}{4} \overset{\times 25}{=} \frac{75}{100}$$

Step 3: Write down 75 with the decimal place 2 spaces from the right (because 100 has 2 zeros);

Answer = 0.75

13.- Convert Decimal to Percent

Just move the decimal point 2 places to the right and add a "%" sign!

So, to convert from decimal to percentage, just **multiply the decimal by 100**, but remember to put the "%" sign so people know it is per 100.

The easiest way to multiply by 100 is to **move the decimal point 2 places to the right**:

From Decimal	To Percent	
0.125 	12.5%	Move the decimal point **2 places to the right**, and add the "%" sign.

Example: Convert 0.65 to percent
Move the decimal point two places: 0.65 -> 6.5 -> 65.
Answer: **0.65 = 65%**

14.- Convert Percent to Decimal

Just move the decimal point 2 places to the left and remove the "%" sign!

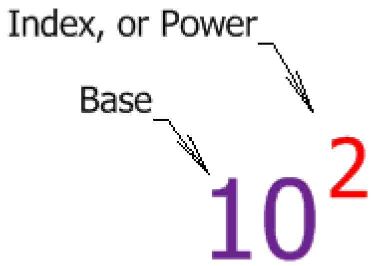
The easiest way to divide by 100 is to **move the decimal point 2 places to the left**.

From Percent	To Decimal	
75% 	0.75	move the decimal point **2 places to the left**, and remove the "%" sign.

Example: Convert 8.5% to decimal
Move the decimal point two places: 8.5 -> 0.85 -> 0.085
Answer: **8.5% = 0.085**

15.- Index Notation and Powers of 10

(Note: *"Index, Power* or *Exponent"* all mean the same thing)
The index of a number shows you **how many times** to use the number in a **multiplication**.



This means **10** × **10**
(**10** is used **2** times in the multiplication)

Example 1: $\mathbf{10^3 = 10 \times 10 \times 10 = 1{,}000}$

- In words: 10^3 could be called "10 to the third power", "10 to the power 3" or simply "10 cubed"

Example 2: $\mathbf{10^4 = 10 \times 10 \times 10 \times 10 = 10{,}000}$

- In words: 10^4 could be called "10 to the fourth power", "10 to the power 4" or simply "10 to the 4"

You can multiply *any* number by itself *as many times as* you want using this notation but powers of 10 have a special use.

Powers of 10

"Powers of 10" is a very useful way of writing down large numbers.
Instead of having lots of zeros, you show how many **powers of 10** you need to make that many zeros

Example: $5000 = 5 \times 1000 = 5 \times 10^3$

- 5 thousand is 5 times a thousand. And a thousand is 10^3. So 5 times $10^3 = 5000$
- Can you see that 10^3 is a handy way of making 3 zeros?

Scientists and Engineers (who often use very big or very small numbers) find it very useful to write numbers this way, such as:

- **9.46×10^{15} meters** (the distance light travels in one year), or
- **1.9891×10^{30} kg** (the mass of the Sun).

It saves them writing down lots of zeros. It is commonly called **Scientific Notation**, or **Standard Form.**
While at first it may look hard, there is an easy "trick":
The index of 10 says **how many places** to move the decimal point to the right.

Example: What is 1.35×10^4?

You can calculate it as: $1.35 \times (10 \times 10 \times 10 \times 10) = 1.35 \times 10000 = 13500$

But it is easier to think "move the decimal point 4 places to the right" like this:

1.35 ➡ 13.5 ➡ 135. ➡ 1350. ➡ 13500.

Negative Powers of 10

Negative? What could be the opposite of multiplying? **Dividing!**
A negative power means **how many times to divide** by the number.
Negatives just go the other way!

Example: $5 \times 10^{-3} = 5 \div 10 \div 10 \div 10 = 0.005$

Just remember:

For negative powers of 10, move the decimal point to the left.

Example: What is 7.1×10^{-3}?
Well, it is really $7.1 \times (^{1}/_{10} \times {}^{1}/_{10} \times {}^{1}/_{10}) = 7.1 \times 0.001 = 0.0071$
But it is easier to think "move the decimal point 3 places to the left" like this:

7.1 ➡ 0.71 ➡ 0.071 ➡ 0.0071

Summary

The index of 10 says how many places to move the decimal point. Positive means move it to the right, negative means to the left.

Example:

	Number	In Scientific Notation	In Words
Positive Powers	5000	5×10^{3}	5 Thousand
Negative Powers	0.005	5×10^{-3}	5 Thousand*ths*

16.- Metric Numbers

In the Metric System there are standard ways of talking about big and small numbers:

- "kilo" for a thousand,
- "mega" for a million,
- and more ...

Example, if a long rope measures **one thousand** meters, it is easier to say it is one **kilo**meter long, and even easier to write it down as 1 **km**.

In that example we used ***kilo*** in front of the word ***meter*** to make "***kilometer***".
And the abbreviation is "***km***" (***k*** for kilo and ***m*** for meter, put together).

Here are some more examples:

Example 2: if we put something on a set of scales and it shows 2000 grams, we can call that 2 kilograms, or even 2 kg.

Example 3: If the doctor wants you to take 5 **thousandths** of a liter of medicine (thousandth=one thousand times smaller), he is more likely to say "take 5 **milli**liters", or write it down as 5mL.

"kilo", "mega", "milli" etc are called "**prefixes**":

Prefix: a word part that can be added to the beginning of another word to create a new word

So, using the prefix "milli" in front of "liter" creates a new word "millilter".
Here we list the prefix for commonly used big and small numbers:

Common Big and Small Numbers

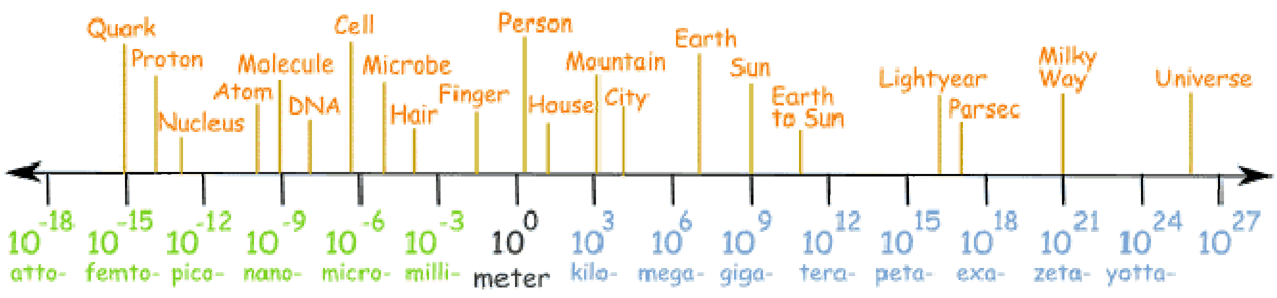
Name	The Number	Prefix	Symbol
trillion	1,000,000,000,000	tera	T
billion	1,000,000,000	giga	G
million	1,000,000	mega	M
thousand	1,000	kilo	k
hundred	100	hecto	h
ten	10	deka	da
unit	**1**		
tenth	0.1	deci	d
hundredth	0.01	centi	c
thousandth	0.001	milli	m
millionth	0.000 001	micro	μ
billionth	0.000 000 001	nano	n
trillionth	0.000 000 000 001	pico	p

Some Very Big, and Very Small Numbers

Name	The Number	Prefix	Symbol
	Very Big !		
septillion	1,000,000,000,000,000,000,000,000	yotta	Y
sextillion	1,000,000,000,000,000,000,000	zetta	Z
quintillion	1,000,000,000,000,000,000	exa	E
quadrillion	1,000,000,000,000,000	peta	P
	Very Small !		
quadrillionth	0.000 000 000 000 001	femto	f
quintillionth	0.000 000 000 000 000 001	atto	a
sextillionth	0.000 000 000 000 000 000 001	zepto	z
septillionth	0.000 000 000 000 000 000 000 001	yocto	y

17.- Advanced Topics!

Here is an illustration of sizes, from the very small (a Quark) to the very large (the known Universe).
The sizes are **in meters** using metric numbers (just add the word "meter" after them, so you get "millimeter" etc):



The numbers (like $\mathbf{10^6}$) are in scientific notation :

Example: $\mathbf{10^6}$ is a 1 followed by 6 zeros: 1 000 000. It is also called a million.
The prefix is mega, so a megameter is a million meters.

Example: $\mathbf{10^{-9}}$ is a 1 moved nine places the other side of the decimal: 0.000 000 001
It is also called a **billionth**.
The prefix is nano, so a nanometer is a billionth of a meter.

Looking at the illustration you can see that a person is about 1 meter in size, a mountain is about$\mathbf{10^3}$ (one thousand) meters in size, and the Sun is about $\mathbf{10^9}$ (one billion) meters in size.

We could also say the Sun is about a "gigameter" in size (it is actually 1.392×10^9 meters, or 1.392 gigameters, or simply 1.392 Gm in diameter).

You can also use Metric Numbers with other measures like seconds, grams and so on.

All Big Numbers We Know

Name	As a Power of 10	As a Decimal
Thousand	10^3	1,000
Million	10^6	1,000,000
Billion	10^9	1,000,000,000
Trillion	10^{12}	1,000,000,000,000
Quadrillion	10^{15}	etc ...
Quintillion	10^{18}	
Sextillion	10^{21}	
Septillion	10^{24}	
Octillion	10^{27}	
Nonillion	10^{30}	
Decillion	10^{33}	
Undecillion	10^{36}	
Duodecillion	10^{39}	
Tredecillion	10^{42}	
Quattuordecillion	10^{45}	
Quindecillion	10^{48}	
Sexdecillion	10^{51}	
Septemdecillion	10^{54}	
Octodecillion	10^{57}	
Novemdecillion	10^{60}	
Vigintillion	10^{63}	

All Small Numbers We Know

Name	As a Power of 10	As a Decimal
thousandths	10^{-3}	0.001
millionths	10^{-6}	0.000 001
billionths	10^{-9}	0.000 000 001
trillionths	10^{-12}	etc ...
quadrillionths	10^{-15}	
quintillionths	10^{-18}	
sextillionths	10^{-21}	
septillionths	10^{-24}	
octillionths	10^{-27}	
nonillionths	10^{-30}	
decillionths	10^{-33}	
undecillionths	10^{-36}	
duodecillionths	10^{-39}	
tredecillionths	10^{-42}	
quattuordecillionths	10^{-45}	
quindecillionths	10^{-48}	
sexdecillionths	10^{-51}	
septemdecillionths	10^{-54}	
octodecillionths	10^{-57}	
novemdecillionths	10^{-60}	
vigintillionths	10^{-63}	